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Structural Similarity in Analogical Transfer

Bachelor's Thesis

by

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0 Abstract

When making an analogy, similarities between the source and the target are of major importance. This study deals with structural similarity and its influence on analogy making in problem solving. Earlier results in this field are meant to be strengthened by an experiment in which the structural similarity between the base and the target is varied. Three different target problems with different structural overlap to the source are presented. It is hypothesised that there is a certain lower limit over which the similarity must lie to enable analogical transfer. The results given fail to define this limit and generally need more experimental support to be reliable. They do underline the importance of structural similarity for analogical transfer, though. Hopefully the problems that occurred here will be considered thoroughly by future experimenters, and coming experiments concerning structural similarity can strengthen existing results and also move a step closer towards grasping the lower limit necessary for transfer.

1 Introduction

Analogical reasoning is one of the most commonly used cognitive processes. It is used in learning and permits transfer across different concepts and situations. It enables us to deal with topics that are completely new to us. In problem solving, analogy can be extremely useful for finding similarities between a problem already solved and one that has not been solved, yet. Furthermore, it has been thought of as a very important part of creativity. (Wilson & Keil, 1999)

When you are faced with a problem you have to solve, it is very useful to remember how you solved a similar problem in the past whose solution you know. If you imagine how you solved complex mathematical problems in school, you will probably recall that you tried to remember what the solution to a similar problem (or various similar problems) was. So, you will use the knowledge from the problem solution you have to solve the new problem.

Analogical transfer can therefore be defined as carrying over information from a known problem, the source (or base), to an unknown one, the target. You retrieved the information you needed to solve the new mathematical problem (target) from the mathematical problem whose solution was already known to you (source).

1.1 Basic Processes Involved in Analogical Reasoning

The process of making an analogy is often divided into four subprocesses. Most commonly these are retrieval, mapping, transfer and evaluation. First of all, when you are confronted with a target problem, a source problem must be retrieved from the knowledge base (you must remember having solved a similar mathematical problem and what the solution to that problem was), then, the source and target problems must be mapped onto each other to define which elements of the source apply to which elements of the target. Once the mapping is complete, knowledge from the source must be transferred to the target. The analogy made must now be evaluated to find out if it is really correct. (e.g., Keane et al., 1994; Falkenhainer et al., 1989; Anderson & Thompson, 1989)

1.2 Different Kinds of Similarity Involved in Analogy Making

Of course the source and the target have to be somewhat similar to each other in order to be suitable for analogical transfer. Two different types of similarity are defined in this context: surface and structural similarity. Surface similarity includes all properties that apply to the surface structure of a problem, the wording, etc.; structural similarity is influenced by the deep underlying structure of the given problems. These structural aspects are defined as causally relevant for achieving the goal, if there are structural dissimilarities this will weaken the analogy. (Holyoak & Koh, 1987; Ross, 1989b) There has been a debate on which type of similarity is more important to analogy making. The general assumption is that surface similarity plays the main role for retrieval, whereas structural similarity is more important for mapping (e.g., Ross 1987, 1989a, 1989b; Holyoak & Koh, 1987, Reed et al., 1990). Mapping and transfer are the most thoroughly researched steps in analogy making. They are also the central processes involved in making an analogy.

1.3 Mapping and Transfer

Mapping is usually more specifically defined as finding corresponding relational structure, that is, mapping the different instances of source and target onto each other in order to transfer knowledge from a part of the source to the part mapped onto that from the target. There are different theories as to how this mapping takes place. It is generally considered to be the major subprocess of analogy making and every computational model includes an implementation of it (e.g., SME: Falkenhainer et al., 1989; ACME: Holyoak & Thagard, 1989a, b; AMBR: Kokinov, 1994a). The structure mapping theory (Gentner, 1983) assumes that objects are only put into correspondence if their roles in the common relational structure correspond.

Higher order structures are preferred, which makes it possible for whole structures to be carried over from the base to the target. Not all researchers consider purely structural aspects important for mapping. Some have suggested that pragmatic factors are important for mapping, as well (Holyoak & Thagard, 1989b). Gick and Holyoak (1980) presume that after some basic propositions and their predicates have been mapped the process becomes a top-down process, guided by the reasoners' expectations.

Generally, there are several different kinds of analogies, for instance spontaneous analogies, in which a source is spontaneously retrieved from memory (Kokinov, 1996). The type of analogy that is often used to study mapping and transfer is forced analogy. Here, the procedure is to provide a source in the form of a helpful example (Novick & Holyoak, 1991). This has proved very useful as spontaneous retrieval seems to be extremely difficult (only about 20% succeed in experiments, Gick & Holyoak, 1980) and this limits the cases in which mapping and transfer occur at all. If a source is provided for making an analogy, spontaneous retrieval becomes unnecessary. This procedure will be used in the following to investigate the influence of structural similarity on mapping and transfer.

An absolutely perfect mapping can only be achieved if the base and target have an identical deep structure, if they are isomorphic. Then, every constituent of the target can be mapped onto a part of the base, which, of course, is only the ideal state. In everyday situations this very seldom appears. It is more likely to have structures without complete structural overlap, cases where the target and the base are not isomorphic. We may find that the target structure not only includes the base structure but some additional features as well (source inclusiveness). This is also possible the other way around where the target structure is only part of the source structure (target exhaustiveness). Often not even one of the two structures can be found in the other and only some features overlap. (For studies concerned with the kind of structural overlap and analogy making, see Schmid et al, 1999; Schmid et al., 2003.)

Another interesting phenomenon in mapping is that one element is not always mapped onto exactly one other element; it is also possible that one element is mapped to many elements or many elements are mapped to one (one-to-many mappings and many-to-one mappings, Holyoak & Thagard, 1989b). In experiments concerning analogical reasoning, the source and target are not always from one domain; there are within-domain analogies as well as analogies between different domains (Vosniadou & Ortony, 1989).

Previously, there have been studies to test to what extent the kind and amount of structural overlap has an influence on mapping (Schmid et al., 2003; Wirth, 1998). They have shown that in order for analogical mapping to be possible a certain amount of structural similarity between the base and the target is necessary.

1.4 Structural Overlap in Mapping and Transfer

The following study was motivated by former studies of Ute Schmid, Joachim Wirth and Knut Polkehn (Schmid et al., 2003; Wirth, 1998). They tried to find out to what extent the kind and degree of structural similarity between source and target influence analogical reasoning. For their studies they used water redistribution tasks which were rather complex in their representation. They found that the amount and kind of structural overlap between the source and the target problem have a major influence on analogical transfer. The performance for a problem solving task, in this case making the correct analogy, does not change much, though, until a certain degree of dissimilarity is reached. Therefore, they state that the suitability of a possible source can clearly be defined by its structural overlap with the target. Possibly the structural similarity must lie above a fixed lower limit to enable analogical transfer. This should be acknowledged by future researchers in this field and future experiments should be planned accordingly as these results clearly show that a certain degree of structural similarity is necessary for correct analogical transfer.

This study is meant first of all to strengthen Schmid et al.'s (2003) results by conducting a similar experiment in a different problem domain. The problem domain was meant to be less complex in its representation and for this reason algebraic word problems will be used. Here, only the degree of structural similarity will be of concern, there will be no tests regarding the kind of structural overlap. The second point of interest is to get a better idea of exactly how much structural overlap is necessary for analogical transfer. Finding the lower limit to the crucial amount of structural overlap can only be possible with much larger data sets and experiments in many different problem domains so the following study does not provide significant results on this point. There have only been a few studies in this field up to now, but there will hopefully be more in the future so that these can be combined to give future researchers a hint as to how much structural similarity is exactly necessary to enable analogical transfer.

In the following a short overview of research on analogy in problem solving will be given (see 2), followed by a more detailed presentation of the problem domain this study is concerned with, including its representation (see 2.2), and how to measure the degree of structural similarity between two problems (see 2.3). An experiment, in which the degree of structural similarity between the base and the target is varied, will be introduced and its results thoroughly discussed after that (see 3).

2 Analogy in Problem Solving

In experiments concerning analogical problem solving, the participants are presented with a novel problem they have to solve. They either know the solution to a similar problem from the past and are tested as to whether they will retrieve it or not, or are given an example problem with a solution (to examine only mapping and transfer as stated in 1.3).

The target problem must be difficult enough that the participants cannot recall the solution directly from memory and can not find a solution by trial and error. Otherwise making an analogy would not be necessary at all (e.g., Newell & Simon, 1972; Hesse, 1991) for finding a solution. Problems that have often been used are word problems in which the participants are confronted with a situation they have to solve. Gick and Holyoak (1980), for instance, presented a solution to Duncker's (1945) tumor problem to participants in which a tumor must be treated with several small lasers, instead of with one large one, in order not to damage the tissue around it but, nevertheless, destroy the tumor. Later the participants were asked to solve a problem of a general who wants to capture a fortress. All ways leading to it are too small for his large army, though. The analogous solution would be to send small parts of the general's army down each road. As Dunbar (2001) pointed out, these problems are all very artificial, and as analogy is very context-sensitive (Kokinov, 1994b; Kokinov & Grinberg, 2001), it might be more useful to find more realistic settings in order to investigate this problem further. Furthermore, most of these experiments were based on analogies between domains and the problems could not be formalized well. This makes it hard to control these experiments even if they are carried out under experimental conditions.

For researching structural similarity, more clearly defined problems must be used. Problems that require a solution consisting of various steps, like the tower of Hanoi, the missionaries and cannibals problem and, as stated above, water redistribution problems have been applied (Schmid et al., 2003). For these problems, initial conditions, legal operations and the goal state are explicitly specified, which is not the case for the problems shown above. Besides, experiments with these problems are solely within domain experiments. These factors are important for applying similarity measures. As Wirth (1998) points out, in most cases it is hard to find a comparable representation for problems from different domains. Due to the many individual steps included in their solution the representations of the problems given above are very complex. For this study algebraic word problems were used as their representation is much less complicated and they are thus easier to deal with (see 2.2).

2.1 Algebraic Word Problems

Quite a lot of the studies concerning algebraic word problems and analogy have been conducted in order to find out more about learning by analogy for educational reasons (e.g., Ross, 1989b). They are very useful for studying analogy making as there is quite a variety of different problems and they are quite easy to manipulate. Furthermore, as will be shown, the representation of an algebraic word problem is not as complicated as it would be for other problems.

In this study algebraic word problems are used to research what effect different amounts of structural overlap between the base and target have on mapping the two correctly and therefore on making the analogy. If we want to examine the influence of the structure of a specific problem, it is necessary to find a representation of this structure that can easily be compared with others.

2.2 Representation of Algebraic Word Problems

The structure of an algebraic word problem is essentially the set of objects and operations, and the relations between them. This structure can also be expressed as a mathematical equation. A word problem, therefore, is isomorphic with its corresponding equation (Nieminen, 1993). Thus, we can concentrate on representing the structure of the problem through the underlying equation. Each word problem's structure will be represented as a tree graph. If we had a word problem with the corresponding equation

$$2x + 3 = 9$$

the graph would look as follows:

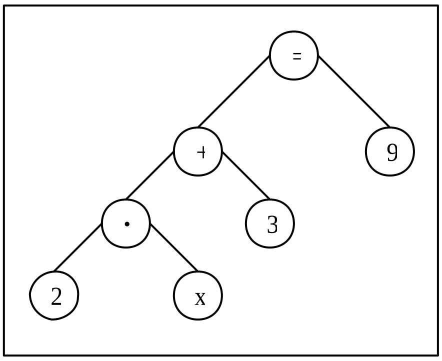

Figure 1: Graph G for the Equation 2x+3=9

This graph representation is very effective as it is clearly structured and also easy to compare with other graphs in order to calculate the structural overlap between two algebraic word problems.

2.3 Measuring the Degree of Structural Overlap

The structural similarity between two graphs G and H is typically measured by calculating the relation between the largest subgraph and the larger of the two graphs (e.g., Schmid et al., 1999, 2003; Bunke & Messmer, 1994). The Structure Mapping Engine applies a similar technique, since finding the common subgraph between the base and target is one of the central concepts used there (Falkenhainer et al., 1989). There are some differences in the exact equation used to calculate this similarity but they only differ slightly, so here the following will be applied.

$$d_{(G,H)} = 1 - \frac{V_{GH} + N_{GH}}{\max(V_G, V_H) + \max(N_G, N_H)}$$

V_{GH} stands for the number of common arcs, whereas N_{GH} represents the number of common nodes. The common number of arcs here stands for all possible arcs, actual arcs and all arcs that would appear if all nodes were connected to each other. V_G and V_H stand for the number of arcs of each individual graph, and N_G and N_H for the number of nodes of the graphs G and H, respectively.

The result of this formula lies between the values zero and one, with $d_{(G,H)}=0$ when the two graphs' structures are isomorphic and $d_{(G,H)}=1$ if they are completely different in structure and have no common subgraph.

Consequently, if we consider the graph in figure 1 as G and the following one (see figure 2) as H, we receive the results shown below.

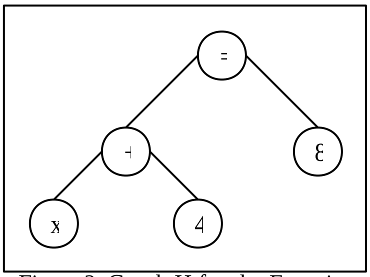

Figure 2: Graph H for the Equation x + 4 = 8

$$V_{GH} = 12 \quad V_G = 42 \quad V_H = 20$$

$$N_{GH} = 4 \quad N_G = 7 \quad N_H = 5$$

So, if you enter these numbers into the formula you get

$$d_{(G,H)} = 1 - \frac{12 + 4}{\max(42, 20) + \max(7, 5)} = 0.67 \quad .$$

Apparently the tree graphs G and H cannot be considered very similar in structure, even though more than half of the larger graph's nodes are shared.

Of course, the results would look slightly different if other similarity measures had been used. However, for all measures based on the comparison between the common graph and the larger of the two, the results would be very close to the one shown above.

3 Experiment

This experiment was designed to test whether the amount of structural overlap between the base and the target has any influence on analogical transfer in problem solving. The problems given were algebraic word problems, mixed problems to be more specific. Three different targets (see 3.2.1) that differed in the amount of structural overlap with the source (for similarity measures see table 2) were given.

3.1 Hypothesis

As this study is meant to strengthen Ute Schmid's and her colleagues' results (see 1.4), results similar to theirs are anticipated. The expectations are that analogical transfer will be hardest for the group that had to solve the problem with little structural overlap to the target problem. The other two groups' outcomes should be a lot better and not too different from each other.

3.2 Method

3.2.1 Material

An identical source problem was given to all participants:

> *Source problem*: Suppose you work in a lab. You need a 15% acid solution for a certain test, but your supplier only ships a 10% solution and a 30% solution. Rather than pay the hefty surcharge to have the supplier make a 15% solution, you decide to mix 10% solution with 30% solution to make your own 15% solution. You need ten liters of the 15% acid solution. How many liters of the 10% and how many of the 30% solution do you need?

Every participant received one of three different target problems that had different levels of structural similarity to the source. There was an isomorphic problem, a problem with high structural similarity, and one with low structural similarity to the source problem. The similarity was measured according to the degree of structural overlap as is shown in 2.3. (See table 1 for similarity measures and appendix, pp. VI-IX, for more details.) The tree graph representation of the source problem is shown in figure 3, it is necessary to measure the degree of structural overlap. You can find the tree graphs for the three target problems in the appendix (pp. VI-IX).

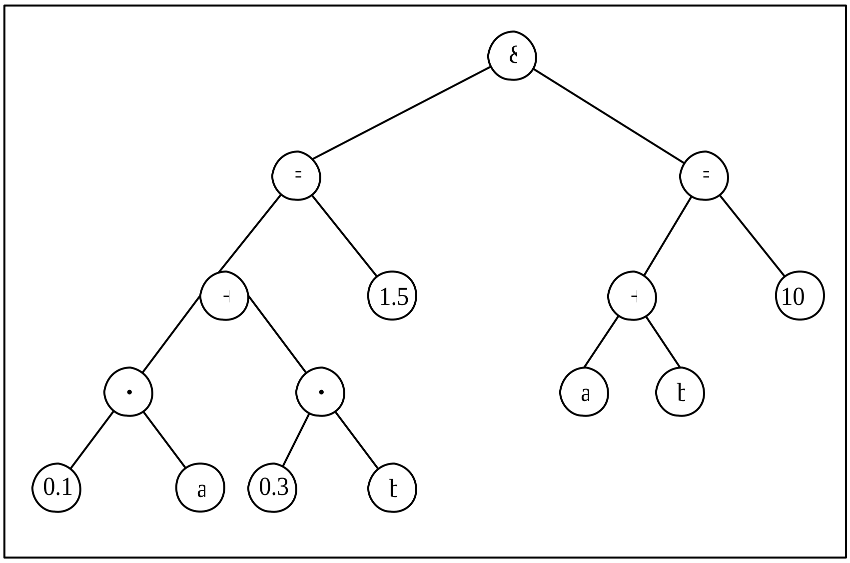

Figure 3: Graph Representation for the Source Problem

Table 1:
Structural Similarity Measures for the Target Problems

Isomorphic problem: d (Source, Target1)= 0
High structural similarity problem: d (Source, Target2)= 0.32
Low structural similarity problem: d (Source, Target3)= 0.73

Table 2:
Target Problems

Isomorphic problem (Target 1): For Christmas the M&M factory plans to produce a special edition consisting of only red and green M&Ms. As the food coloring has various different prices, one kilogram of green M&Ms costs 1 Euro whereas one kilogram red M&Ms costs 2 Euros. How many kilograms of each color must be produced to receive 9 kilograms of Christmas M&Ms that cost exactly 13 Euros?
High structural similarity problem (Target 2): The M&M factory plans to produce a special edition once again consisting of only red and green M&Ms for Christmas this year. The costs for this special edition last year were 24 Euros per 10 kilograms. As the food coloring has various prices, one kilogram of green M&Ms costs 1 Euro whereas one kilogram red M&Ms costs 3 Euros. The M&M producer wants to improve his production costs and save exactly ¾ of the costs of the year before. How many kilograms of each individual color of M&Ms must be combined to receive nine kilograms that match his price expectations?
Low structural similarity problem (Target 3): For Christmas the M&M factory plans to produce a special edition consisting of only red and green M&Ms. However the M&M producer hears that the Smarties producer is also planning to produce such an edition. He will combine his Smarties-mixture from seven kilograms of green and three kilograms of red Smarties and will pay 5.32 Euros per kilogram. The M&M producer is happy as his special edition with the same proportions costs 1.20 Euros less. He immediately tells one of his workers, who asks him how much one kilogram of M&Ms would cost if it consisted of equal amounts of green and red M&Ms. The producer answers 6 Euros. How much does one kilogram of green M&Ms cost and how much does one kilogram of red M&Ms cost?

There was a handout consisting of four sheets of paper that every participant received. It consisted of an instruction, a description of the source problem, the target problem and a questionnaire. (See source and target problems above and appendix for details and original German problems.)

To solve the given word problems a system of two equations with two variables was necessary. This also applies for the source problem, for which the system and its solution were provided and explained (see source solution in appendix, pp. IV & V).

The questionnaire included questions concerning the age and degree program of the participants and similar issues, but more importantly concentrated on mathematical knowledge and the way the target problem had been solved, asking the participants to map the source to the target problem. For mathematical knowledge, participants were asked to rate how familiar they were with this kind of task. An important question was also, if they found the example problem helpful or could have done equally well without. They were also asked how much more time they would have needed to solve the target problem if they had not see the solution to the source problem before. Figure 4 shows you a translation of the questions concerning mapping, the whole questionnaire can be found in the appendix (pp. X & XI).

Please fill in which element from the second word problem corresponds to the one given from the first word problem.

10% acid solution - ______________________

30% acid solution - ______________________

15% acid solution - ______________________

Number of liters of 10% acid solution (a) -

____

Number of liters of 30% acid solution (b) -

____

Figure 4: Question for Judging Correct Mapping

3.2.2 Participants

The participants were thirty-nine students taking part in an English grammar course at the University of Hildesheim. They were studying in the degree programs International Information Management, International Technical Communication, and International Communication and Translation and were mostly in their second semester of studies. They were aged between twenty and thirty with the average age lying at 22 and a very high percentage of participants was female (83%). Most of them were native speakers of German (58%), 14% did not list their mother tongue (these will be treated as non-native German speakers in the following) and all other participants were foreign exchange students. Three participants had to be excluded as they did not give any answers apart from their age and mother tongue.

3.2.3 Procedure

The experiment took place in a lecture hall at the University of Hildesheim in an early morning class on English Grammar. All participants were given a handout as described above. This handout included written instructions, which were repeated by the experimenter. The source problem and its solution were presented using an overhead projector and were explained by the experimenter. The participants could follow this in their handouts.

Then the participants were asked to solve the target problem and to answer the questions on the questionnaire once they had accomplished this task. The participants had about fifteen minutes to work on the target problem. After this time most of the participants had finished solving the problem and turned to the questionnaire. The others were asked to stop where they were and to turn to the questions on the next sheet. All solutions were collected by the experimenter following this.

3.3 Results

First of all, it must be mentioned that very few participants managed to solve the problems correctly. Only 12 of the 36 participants received correct results for the word problems and only seven of these also had their mapping correct. One person was correct on the mapping but got wrong results for the word problem due to some miscalculations.

3.3.1 General Remarks

Overall, the results gathered were not very convincing, the group of participants was definitely too small and therefore, none of the desired results were statistically significant. Nevertheless, the results will be thoroughly discussed here. They must be understood more as results of a test experiment and could not be used scientifically on their own. They can be useful, though, as a guideline and prediction of what results an experiment with more participants would be likely to produce. Particularly taken together with some results from previous experiments, especially those of Schmid and her colleagues (2003), these results may at least point in a certain direction. The results will be discussed critically after they are presented.

3.3.2 Raw Data

As the number of correct answers was very small, we will first of all look at the exact numbers before analyzing them statistically. The numbers in table 3 show that the low similarity problem was the most difficult. There is not a big difference between the isomorphic problem and the high similarity problem, as was predicted. Mapping and transfer is hardest when there is low structural similarity between the base and the target, whereas – to a certain degree of structural overlap – mapping is a lot easier. Table 3 also shows that most participants with correct mapping also got correct solutions. As correct mapping is a good indicator for understanding the analogy, this shows that most participants that actually used analogical reasoning to solve the target problems solved them successfully. Another indicator for having used analogy to solve the problem is stating that one found the source problem's solution helpful for solving the target problems. Only 12 of the 22 German participants and only 19 participants in total found the source helpful (see question 5. in questionnaire, appendix, pp. X & XI).

Table 3:
The Exact Numbers and Percentages of Correct Answers

Mother Tongue	German			All			Non-German		
Target problem	Iso-morphic	High similarity	Low similarity	Iso-morphic	High similarity	Low similarity	Iso-morphic	High similarity	Low similarity
Solution correct	4 *50%*	4 *67%*	1 *14%*	5 *42%*	6 *50%*	1 *8%*	1 *25%*	2 *33%*	-
Mapping correct	2 *25%*	3 *50%*	1 *14%*	2 *17%*	5 *42%*	1 *8%*	-	2 *33%*	-
Solution & Mapping correct	2 *25%*	3 *50%*	-	2 *17%*	5 *42%*	-	-	2 *33%*	-
All	8	6	7	12	12	12	4	6	5
	21			36			15		

3.3.3 Statistical Analysis

For the statistical analysis only participants with German as their mother tongue can be considered, as there is no certainty that all non-native speakers understood the problem as well as the native speakers. In other experiments all participants with an incorrect mapping were excluded (Schmid et al., 2003; Wirth, 1998). This procedure is very reasonable as one can only be sure that these participants actually understood the analogy. This procedure unfortunately cannot be used here, however, as the number of participants is so small that it would not be advantageous to exclude even more. Additionally, the only significant result that could be obtained was the correlation between the solution and mapping ($r(X,Y)=0.5173$; $p=0.016$). Almost all participants that got correct results for their mapping also gave a correct solution, and almost all participants who failed to give a correct solution also had problems with the mapping. Therefore, for this analysis it seems reasonable not to concentrate solely on answers with correct mappings.

To ensure that only participants that really used analogical reasoning for solving the problems will be considered here, all participants that did not find the solution to the source problem helpful will be excluded. This leaves only 12 participants for analysis (of these 12 only 4 were correct on their mappings). This number is surely too small to give reliable results, but the findings will nevertheless be presented here as they can definitely be useful for following experiments in this field.

An analysis of variants (ANOVA) was carried out for the target type and mapping as main factors and also for the target type and the solution, but no statistically significant result could be obtained. (The one-way ANOVAs for the target type and the mapping gave $F=2.79$ and $p=0.12$ and $MS=0.52$; for target type and solution $F=1.09$ and $p=0.38$ and $MS=0.27$.)

3.3.4 Mathematical Knowledge

The participants had to answer questions concerning their prior knowledge of word problems of the kind given in this experiment. In one question they had to rate how familiar they were with these problems (see question 7 in the questionnaire, appendix, pp. X & XI). On the questionnaire they could choose between "very familiar", "familiar", "not very familiar" and "I have never seen such a problem before". The results for this question are shown in table 4. It must also be said that three participants did not give their familiarity ratings at all. Another question was whether the participants could have solved the target problem without seeing the solution to the source problem (see question 6 in the questionnaire, appendix, pp. X & XI). Fifteen of the participants stated that they could have solved the target problem without seeing the source (among these 5 had to solve the isomorphic target problem, 4 got the high similarity target and 6 received the low similarity target). Interestingly, only eight of these participants really were able to give a correct solution. Sixteen participants said they would not have been able to solve the problem without the given example. The third question concerning familiarity with problems such as those given was if the participants would have needed more time for solving the target problem if they had not seen the source (see question 8 in the questionnaire, appendix, pp. X & XI). It was assumed that the participants would answer this question by saying they needed a lot more time if they were insecure in this field. Evidently, this question again gives an insight into whether or not the participants actually used analogies here. Many participants (12 of the 36) said they would have needed a little more time, just about as many participants said they would have needed a lot more time (11 of the 36) and four participants said it did not make any difference to them. Five of the 36 participants stated that they wouldn't have been able to solve the target problem without the source problem (and all of these failed to give a correct solution).

Table 4:
Prior Mathematical Knowledge: Familiarity Ratings

Familiarity ratings	Very familiar	Familiar	Not very familiar	Never seen such a problem
all	2	12	18	1
German	-	7	13	-

As stated above (see 3.3.3), for the analysis of analogical problem solving in this experiment quite a few participants must be excluded. In this case some severe problems occur with only twelve participants. Among these participants were only those that stated they were familiar or not very familiar with such a problem and additionally, all participants that stated they were not very familiar with such problems had to solve an isomorphic target problem. Under these conditions a proper analysis is impossible. As a result, all German participants had to be included in the analysis. Factorial ANOVAs were carried out to find the effects of mathematical knowledge and target type on correct solutions. The factorial ANOVA for the target type and the familiarity measures (results from question 7 in the questionnaire, see appendix, pp. X & XI, and table 4) as main factors taken together with the solutions to the target problems gave: $F=0.31$, $p=0.74$ and $MS=0.058$. It can, however, be said that participants who rated their familiarity higher also performed better on the problem solving task, even if these results are not significant. Not very surprisingly, this is also true for the question whether the participants could have solved the target problem without seeing the solution to the source problem. Here the participants that gave yes as an answer did better than those that answered with no. For the additional time needed in absence of the example the results are not as clear. Unfortunately no significant interaction between target type and mathematical knowledge could be found.

3.3.5 Differences Between Native and Non-Native Speakers

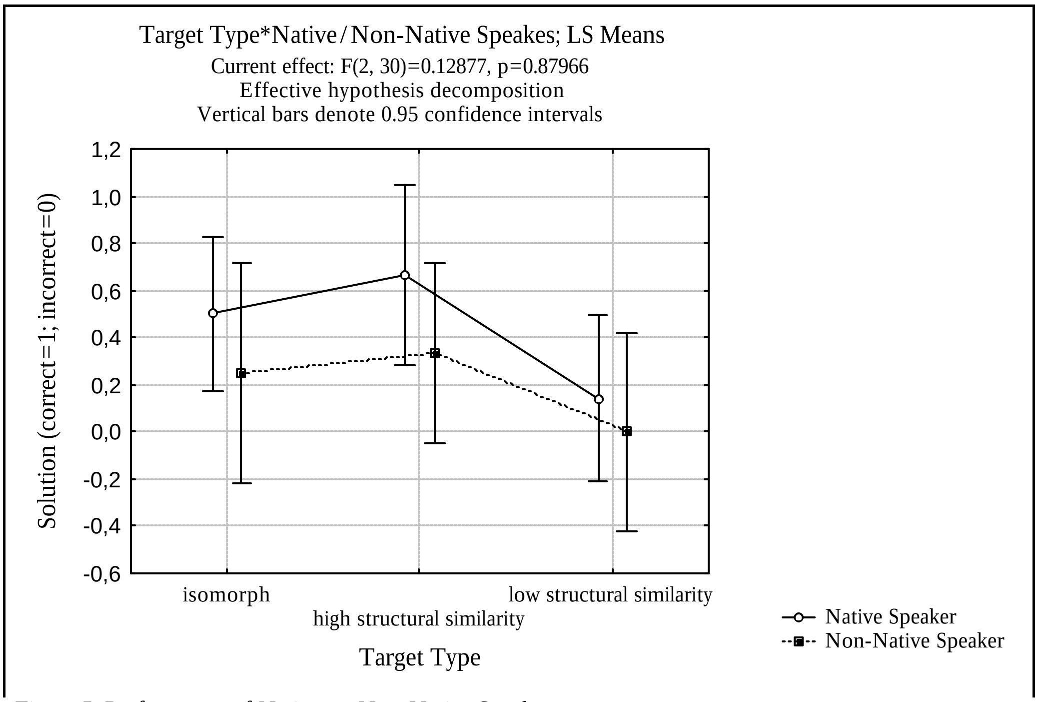

Figure 5: Performance of Native vs. Non-Native Speakers

A factor that is not directly concerned with the main problems of this thesis but is nevertheless very interesting is cultural differences. Quite a lot of foreign students participated in this study which enables us to take a closer look at their performance in contrast to that of the German students. No interaction between the main factors target type and native / non-native speakers could be found. A factorial ANOVA was carried but gave no statistically significant results: $F=0.13$, $p=0.88$, $MS=0.027$ (also see figure 5).

At least it can be said that native speakers performed better on solving the target problems for all target types as figure 5 shows. This is not a very surprising result as it is very likely that some of the foreign students participating in this experiment had some language problems to cope with which surely made the task more difficult for them. In the past mathematical knowledge has been shown to be a good predictor of analogical transfer (Novick & Holyoak, 1991). The given results showed no significant correlation between familiarity ratings (see question 7. on appendix, pp. X & XI) and the mapping results ($r(X,Y)=0.435$, $p=0.55$), though.

3.4 Discussion

Unfortunately the experiment failed to show that the degree of structural overlap between the base and the target is very important for mapping them. It would have been very interesting to find a border line from which point on it is a lot harder to map the base and the target, however, this could not be achieved.

The absolute numbers at least show a tendency towards the desired results. It is clear that the target problem with little structural similarity was hardest to solve and there is no great difference between the other two target problems. If there had been more participants these results would probably have been a lot clearer. In former studies the procedure for excluding participants has been even harder (see also 3.3.3). To ensure that all analyzed participants really used analogy to aid them in solving the given problems only participants that could map the problems correctly were considered (Schmid et al., 2003). This is surely the preferred procedure, but here, this would not have given us any results to discuss. For this experiment all participants were asked whether they found the source problem and its solution helpful for solving the target problems. If they could answer this question with yes, they were considered as having used analogical reasoning. Unfortunately this procedure again left only a very small number of participants to deal with.

Generally there were fewer correct results than had been expected. All participants were students and it was assumed that they had all learned how to solve problems such as those given in school. Only 34% of the participants managed to give a correct solution, however. For mapping, the results were worse than expected as well, merely 22% gave correct answers here. Supposedly these results would have been better if the participants had been more motivated. A lot of them only gave some or even no answers on the questionnaire and also gave the impression of being unwilling to solve the mathematical problems (some of them even stated an aversion towards mathematical problems together with their solutions). No answers or incomplete ones had to be counted as incorrect answers even though in many cases participants are presumed capable of giving correct answers. At least some of these effects could have been dealt with better in a larger group.

If this experiment had been carried out with more participants, the results would likely have been more promising. More motivation or a completely different group of participants that had some interest in taking part in the experiment could have been useful, too. The abilities of the participants were overestimated; more correct results for the isomorphic and high structural overlap problems were expected. Hopefully a similar experiment can be conducted to clarify the given results.

4 Final Discussion

Several experiments have shown that there is a close correlation between the degree of structural similarity and the correctness of mapping and transfer from the base to the target (Schmid et al., 2003). Unfortunately this experiment failed to truly strengthen these results. Further research must show how far structural similarity influences analogical transfer. It would be especially interesting to get a better idea of how much structural overlap there must be between the source and the target to ensure analogical mapping.

Future experiments in this field should definitely be carried out with larger groups of participants. Additionally, the mathematical knowledge was very hard to evaluate. The participants had different prior knowledge and therefore performed very differently. Such experiments are so dependent on mathematical knowledge that it might be reasonable to test the participants' knowledge prior to the experiment and then to design the problems, accordingly. Studies concerning experts and novices making analogies have shown major discrepancies between their behaviors (Novick, 1988a, b). Experts tend to represent a problem in a more abstract fashion than novices do. Perhaps the representation given in the source (in form of the equation) was not equally well understood. To fully control these factors it might be effective to carry out this experiment with school children that have no prior knowledge of this type of problem. In this particular case, the most convenient stage would be to pick students that have learned to solve equations with one variable but are not yet familiar with solving a system of equations with two variables. Then the participant's representation of the problem should be very similar and following this their results will presumably be more consistent. Additionally, this procedure rules out participants that can already solve the problem without the example (and, therefore, without making an analogy), as none of the school children will have solved such a problem before.

Even though the results presented showed no significance, they point into the same direction as those of Schmid et al. (2003) do. It cannot be argued that structural similarity has an influence on analogical mapping and transfer, and it also has become clear that this influence is quite a major one. In future experiments regarding analogy making this surely must be taken into account. Structural similarity measures between the base and target should always be made to ensure that the conditions for studying analogical transfer are good. Understanding how much structural similarity is necessary could also be advantageous in the educational field. Knowing the degree of structural overlap needed could be of aid in designing examples for textbooks and lessons. Computational models of analogy making could also profit from adding a structural similarity measure. This would probably enable them to simulate human behavior more reliably.

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APPENDIX

In the appendix you will find all the problems used in the experiment. In each case the original German version is presented first, followed by a translation, except for those cases where the English is given in the main body of this thesis.

Source Problem

Sie sind ein Wissenschaftler in einem Labor. Sie brauchen für einen speziellen Test eine 15%-ige Säurelösung, aber Ihr Lieferant bietet nur 10%-ige und 30%-ige Lösungen an. Deshalb beschließen Sie die 15%-ige Lösung aus der 10%-igen und der 30%-igen Säurelösung herzustellen. Sie brauchen genau 10 Liter von der 15%-igen Säurelösung.
Wie viele Liter der 10%-igen Lösung und wie viele Liter der 30%-igen Lösung benötigen Sie dazu?

(for the translation into English see 3.2.1, p. 11)

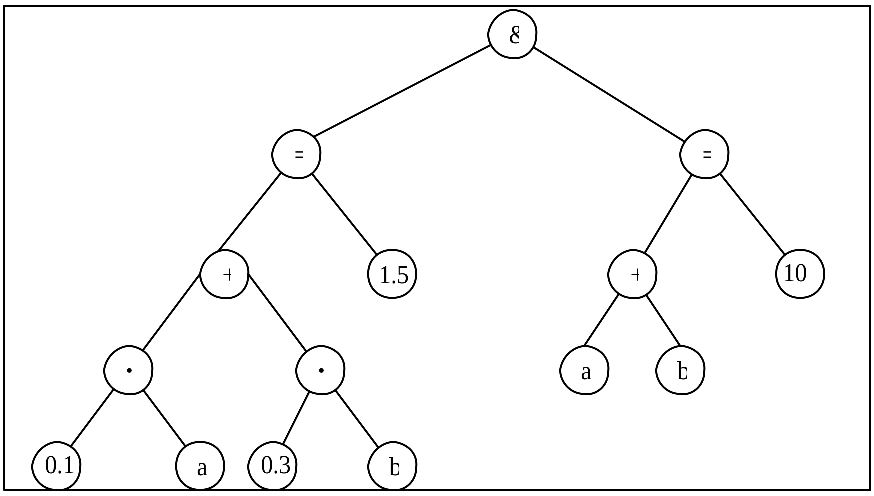
The equations belonging to this problem are the following:

$$0.10a + 0.30b = 1.5 \quad \& \quad a + b = 10$$

And this gives us the results:

$$a = 7.5 \text{ liters} \;\&\; b = 2.5 \text{ liters}$$

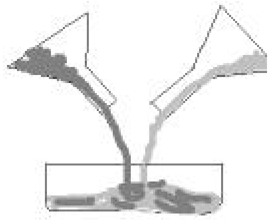
The graph representation is as follows (see also 3.2.1, p. 12).



Now the original solution to the source problem and its translation will be given, as each participant received it in the experiment.

Sie sind ein Wissenschaftler in einem Labor. Sie brauchen für einen speziellen Test eine 15%-ige Säurelösung, aber Ihr Lieferant bietet nur 10%-ige und 30%-ige Lösungen an. Deshalb beschließen Sie die 15%-ige Lösung aus der 10%-igen und der 30%-igen Säurelösung herzustellen. Sie brauchen genau 10 Liter von der 15%-igen Säurelösung.
Wie viele Liter der 10%-igen Lösung und wie viele Liter der 30%-igen Lösung benötigen Sie dazu?

1. Aus dem oberen Text lassen sich zwei Gleichungen herausfinden, die beim Lösen der Aufgabe helfen.
Wir haben im Labor 10%-ige und 30%-ige Lösung und wollen 15%-ige Lösung bekommen. Wir wollen also wissen, wie viele Liter wir von den vorhandenen Lösungen brauchen um 10 Liter 15%-ige Lösung zu bekommen.
Wenn a die Anzahl der gebrauchten Liter 10%-ige und b die Menge der 30%-igen Lösung ist, bekommt man die Gleichung:



0,10a + 0,30b = 0,15 · 10

Die Menge beider Lösungen muss dann 10 ergeben, denn es werden genau 10 Liter der 15%-igen Lösung gebraucht:

a + b = 10

2. Die beiden Gleichungen sind ausreichend, um zur Lösung des Problems zu kommen.
Man löst sie indem man eine Gleichung nach b (bzw. a) auflöst und dies dann in die andere einsetzt.

Wir lösen die zweite Gleichung nach b auf:
a + b = 10
⇨ **b = 10 – a**

und setzen dann 10 – a in die erste Gleichung für b ein:
0,10a + 0,30 · (10 – a) = 0,15 · 10

und lösen diese:
⇨ 0,1a + 0,3 · (10 – a) = 1,5
⇨ 0,1a + 3 – 0,3a = 1,5
⇨ - 0,2a + 3 = 1,5 | – 3
⇨ - 0,2a = - 1,5 | · (-1)
⇨ 0,2a = 1,5 | · 5
⇨ **a = 7,5** .

Dies könnte man auch als Formel ausdrücken. Demnach ist

$$a = \frac{0{,}15 \cdot 10 - 0{,}30 \cdot 10}{0{,}10 \cdot 0{,}30} = \frac{-1{,}5}{-0{,}2} = 1{,}5 \cdot 5 = 7{,}5$$

Wir können nun einfach 7,5 für a in eine der beiden Gleichungen einsetzen und somit auch b erhalten.

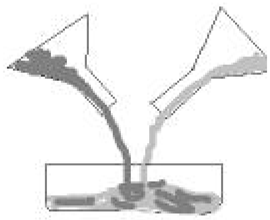
7,5 + b = 10 ⇨ **b = 10 – 7,5 = 2,5**

Unsere Ergebnisse sind also:

a = 7,5 und b = 2,5

Das bedeutet also, dass 7,5 Liter der 10%-igen und 2,5 Liter der 30%-igen Lösung benötigt werden um 10 Liter 15%-ige Lösung herzustellen.

Suppose you work in a lab. You need a 15% acid solution for a certain test, but your supplier only ships a 10% solution and a 30% solution. Rather than pay the hefty surcharge to have the supplier make a 15% solution, you decide to mix 10% solution with 30% solution to make your own 15% solution. You need ten liters of the 15% acid solution. How many liters of the 10% and how many of the 30% solution do you need?



1. From the text above you can deduct two equations that help with solving the problem.
In the lab we have 10% and 30% acid solution but we want to get 15% acid solution. So, we want to know how many liters of the available solutions we need to receive 10 liters of 15% acid solution.
Let a be the amount of liters of 10% acid solution required and b the amount of 30% acid solution. That gives us the following equation:

0.10a + 0.30b = 0.15 · 10

The amounts of both solutions must add up to 10, as exactly 10 liters of the 15% solution are needed:

a + b = 10

2. These two equations are sufficient to find a solution of the problem.
This is done by resolving one equation for b (or a) and then entering the results in the second equation.

We resolve the second equation for b:
a + b = 10
⇨ **b = 10 – a**

and fill in 10 –a for b in the first equation:
0.10a + 0.30 · (10 – a) = 0.15 · 10

and resolve this:
⇨ 0.1a + 0.3 · (10 – a) = 1.5
⇨ 0.1a + 3 – 0.3a = 1.5
⇨ - 0.2a + 3 = 1.5 | – 3
⇨ - 0.2a = - 1.5 | · (-1)
⇨ 0.2a = 1.5 | · 5
⇨ **a = 7.5** .

This can also be expressed with a formula. This is, accordingly,

$$a = \frac{0.15 \cdot 10 - 0.30 \cdot 10}{0.10 \cdot 0.30} = \frac{-1.5}{-0.2} = 1.5 \cdot 5 = 7.5$$

Now we can simply replace a with 7.5 in one of the two equations and also receive b.

7.5 + b = 10 ⇨ **b = 10 – 7.5 = 2.5**

Therefore, our solutions are:

a = 7.5 and b = 2.5

This means that 7.5 liters of the 1% acid solution and 2.5 liters of the 30% solution are required to produce 10 liters of 15% acid solution.

Isomorphic Target Problem

Zu Weihnachten sollen in der M&M Fabrik für eine Sonderaktion nur rote und grüne M&Ms hergestellt werden. Da der Farbstoff unterschiedlich teuer ist, kostet ein Kilo grüne M&Ms 1 Euro und ein Kilo rote 2 Euro. Wie viele Kilo von jeder Farbe müssen hergestellt werden um 9 Kilo zu bekommen, die genau 13 Euro kosten?

(for the translation into English see 3.2.1, p. 13)

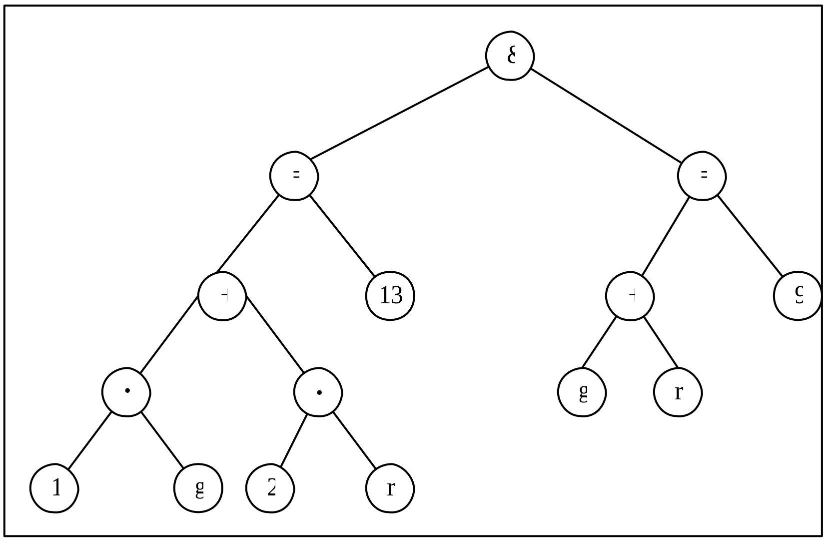
The equations belonging to this problem are the following:

1g + 2r = 13 & g + r = 9

And this gives us the results:

g = 5 kilograms & r = 4 kilograms

The graph representation looks as follows.



Structural similarity to the source problem:

$$d_{(S,T)} = 1 - \frac{210 + 15}{\max(210, 210) + \max(15, 15)} = 0 \quad .$$

High Structural Similarity Target Problem

Zu Weihnachten soll es in diesem Jahr in der M&M-Fabrik wieder eine Sonderaktion mit dem Verkauf von Packungen mit einer Mischung aus nur roten und grünen M&Ms geben. Die Herstellungskosten für eine solche Mischung betrugen im letzten Jahr 24 Euro für zehn Kilo. Da der Farbstoff unterschiedlich teuer ist, kostet ein Kilo grüne M&Ms in diesem Jahr 1,00 Euro und ein Kilo rote 3,00 Euro. Der M&M-Hersteller möchte seine Herstellungskosten gegenüber dem Vorjahr verbessern und genau ¾ der Kosten vom Vorjahr zahlen. Wie viele Kilo von jeder Farbe müssen gemischt werden um neun Kilo zu bekommen, die seiner Preisvorstellung entsprechen?

(for the translation into English see 3.2.1, p. 13)

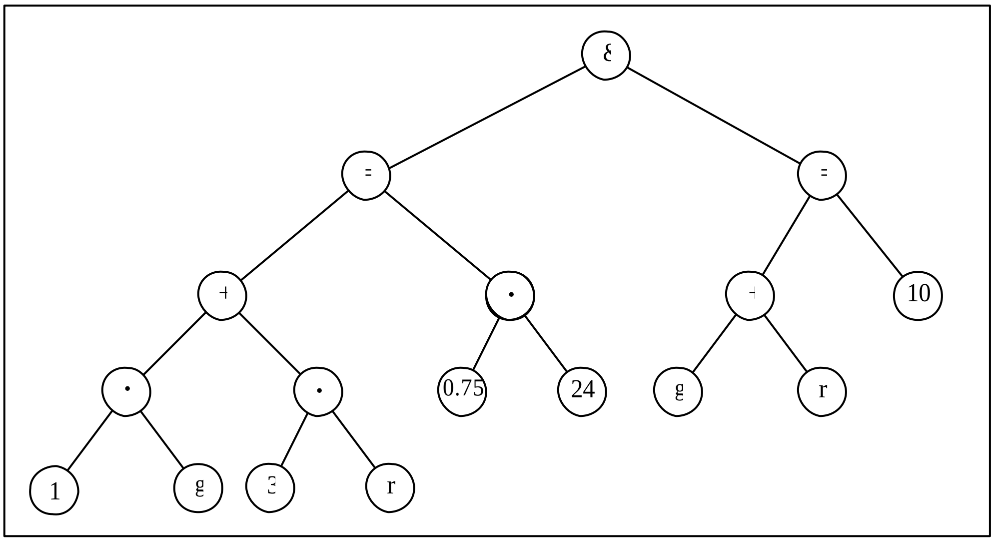
The equations belonging to this problem are the following:

$$1g + 3r = 3/4 \cdot 24 \;\; \& \;\; g + r = 10$$

And this gives us the results:

g = 4 kilograms & r = 6 kilograms

The graph representation is as follows.



Structural similarity to the source problem:

$$d_{(S,T)} = 1 - \frac{182 + 14}{\max(210, 272) + \max(15, 17)} = 0.32 \quad .$$

<u>Low Structural Similarity Target Problem</u>

Zu Weihnachten sollen in der M&M Fabrik für eine Sonderaktion nur rote und grüne M&Ms hergestellt werden. Der M&M-Hersteller hört aber, dass der Smartie Hersteller auch solch eine Aktion plant. Er wird seine Smarties-Zusammensetzung aus je sieben Kilo grünen und drei Kilo roten herstellen und dafür 5,32 Euro pro Kilo zahlen. Der M&M Verkäufer freut sich, denn ihn kostet die gleiche Gesamtmenge bei gleicher Zusammensetzung 1,20 Euro weniger.
Dies erzählt er sofort einem seiner Angestellten, der ihn fragt wie viel ein Kilo kosten würde, wenn es zu gleichen Teilen aus roten und grünen M&Ms zusammengesetzt wäre. Der Hersteller antwortet 6 Euro.
Wie viel kostet ein Kilogramm grüne M&Ms und wie viel ein Kilo rote M&Ms?

(for the translation into English see 3.2.1, p. 13)

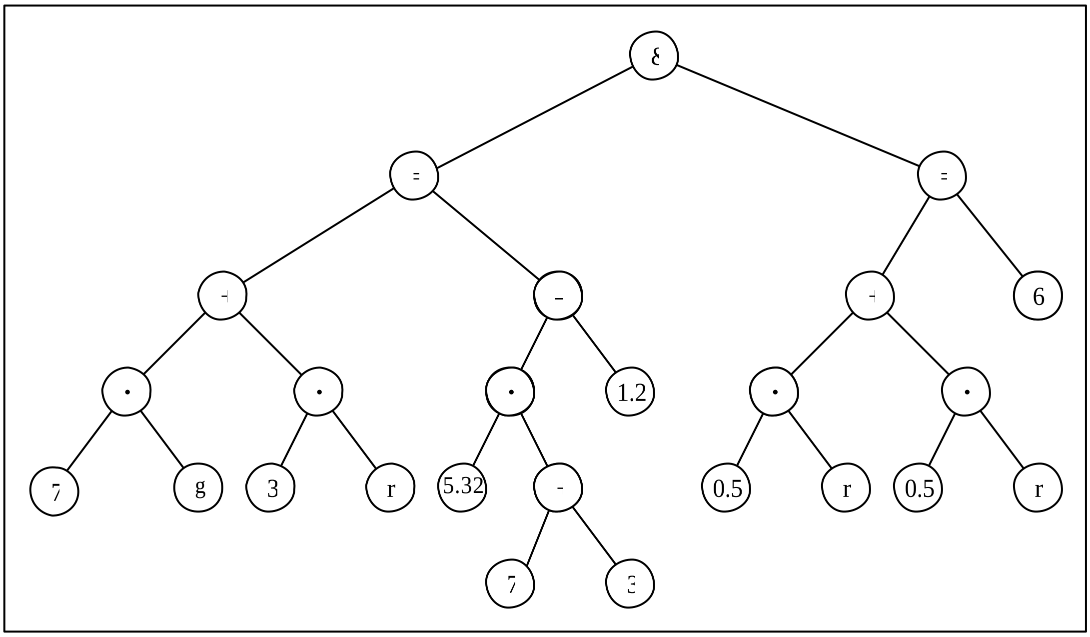
The equations belonging to this problem are the following:

$$\mathbf{7g + 3r = 5.32 \cdot (7 + 3) - 1.20 \quad \& \quad \tfrac{1}{2}\,g + \tfrac{1}{2}\,r = 6}$$

And this gives us the results:

g = 4 Euros & r = 8 Euros

The graph representation is as follows.



Structural similarity to the source problem:

$$d_{(S,T)} = 1 - \frac{132 + 12}{\max(240, 506) + \max(16, 23)} = 0.73 \quad .$$

Questionnaire

1. Bitte geben Sie Ihr Alter an: ________________ .

2. Geschlecht: ? männlich ? weiblich

3. Studiengang: _________________________

4. Semester: _________

5. Fanden Sie die erste Aufgabe hilfreich beim Lösen der zweiten Aufgabe?
   ? Ja ? Nein

6. Hätten Sie die zweite Aufgabe lösen können ohne vorher die Lösung der ersten Aufgabe gesehen zu haben?
   ? Ja ? Nein

7. Wie vertraut sind Sie mit diesem Aufgabentyp?
   ? sehr vertraut
   ? vertraut
   ? wenig vertraut
   ? ich habe nie eine solche Aufgabe gesehen

8. Wie viel mehr Zeit hätten Sie zum Lösen der zweiten Aufgabe gebraucht wenn Sie die erste nicht gesehen hätten?
   ? gar keine
   ? etwas
   ? viel
   ? ich hätte die 2. Aufgabe nicht lösen können

9. Bitte geben Sie an, welches Element der zweiten Aufgabe mit dem angegebenen der ersten Aufgabe korrespondiert.

   10%-ige Säurelösung - ________________________

   30%-ige Säurelösung - ________________________

   15%-ige Säurelösung - ________________________

   Anzahl der Liter 10%-iger Säurelösung (a) - _____________________________________

   Anzahl der Liter 30%-iger Säurelösung (b) - _____________________________________

Vielen Dank für die Teilnahme an diesem Experiment!

1. Please give your age: _______________ .

2. Sex: ? male ? female

3. Degree Program: ________________________

4. Semester: ________

5. Did you find the solution to the first problem helpful for solving the second problem?
   ? Yes ? No

6. Could you have solved the second question without having seen the solution to the first?
   ? Yes ? No

7. How familiar are you with problems such as the ones given?
   ? very familiar
   ? familiar
   ? not very familiar
   ? I have never seen such a problem before

8. How much more time would you have needed to solve the second problem if you had not seen the solution to the first?
   ? no time at all
   ? a little time
   ? a lot of time
   ? I would not have been able to solve the second problem

9. Please fill in which element from the second word problem corresponds to the one given from the first word problem.

   10% acid solution - ______________________

   30% acid solution - ______________________

   15% acid solution - ______________________

   Number of liters of 10% acid solution (a) - ___________________________________

   Number of liters of 30% acid solution (b) - ___________________________________

Thank you very much for participating in this experiment!

Hiermit erkläre ich, Sarah Linnes Irwin, die vorliegende Arbeit „Structural Similarity in Analogical Transfer“ selbstständig verfasst zu haben und keine anderen Quellen oder Hilfsmittel als die angegebenen verwendet zu haben.

Osnabrück, den 30. November 2003

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